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STUDIES ON THE VORTICES IN THE ATMOSPHERE OF THE EARTH.

By Prof. FRANK H. BIGELOW.

I.—THE APPLICATION OF THE THEORY OF VORTEX MOTION TO THE FUNNEL-SHAPED WATERSPOUT AT COTTAGE CITY, AUGUST 19, 1896.

INTRODUCTORY REMARKS.

The purpose of the series of papers on the thermodynamics of the atmosphere, which appeared in the MONTHLY WEATHER REVIEW during the year 1906, was to indicate the distribution of the masses of air of different temperatures in the neighborhood of the axes of cyclones, anticyclones, and a typical waterspout, and to develop the formulas which are useful in discussing the energy contained in them, available in the restoration to a thermal equilibrium under the action of gravity. When a sheet of relatively cold air overlies a sheet of relatively warm air there will be an interchange of position, and in changing places there will be a development of certain stream lines which it is important to understand as fully as possible. Such a distribution of stratified air is an efficient cause of the formation of the vortices popularly called the tornado, the waterspout, and the hurricane. There are two types of such vortices prevailing in the earth's atmosphere, each of which is represented in the Cottage City waterspout of August 19, 1896.[1] The first type is seen in the second appearance, as on Chamberlain's photograph, 2d A, and the second type is found in the third appearance, as on Chamberlain's photograph, 3d A. It will be shown that the St. Louis tornado, May 27, 1896, and the De Witte typhoon, August 1–3, 1901, belong to the first, or dumb-bell, type, while many small funnel-shaped tornadoes belong to the second type. These typical examples will be fully worked out, and the velocities, radial (u), tangential (v), and vertical (w), computed, together with the various relations connecting them together. When two masses of air of different temperatures lie side by side the stream lines which are generated in the thermal flow are of a very different type from those of the preceding cases, in so far as the cyclone represents a pure vortex motion of any type. The general vortices in the earth's atmosphere or other atmospheres belong to still other classes. These vortices were summarized on pages 512, 513, of the International Cloud Report,[2] and in the MONTHLY WEATHER REVIEW, January, 1904;[3] but in this present series of papers an attempt will be made to find the constants and the velocities existing in these specific examples. The final step in the solution of this class of problems will consist in correlating the observed temperature and pressure conditions with these computed velocities. It will be important to develop the computations in detail, so that meteorologists may be able to discuss the circulations of the air as practical examples of the interchange of energy in the atmosphere. The knowledge already attained regarding the temperatures in the earth's atmosphere justifies us in making an effort to advance these fundamental problems in dynamical meteorology. It seems to me quite probable that the best way to determine the physical constants belonging to the sun's atmosphere, i. e., the specific heats and the temperature gradients, will be by utilizing the visible surface velocities of the solar vortex, which is a function of the same.

THE FORMULAS OF VORTEX MOTION.

The subject of vortex motion applicable to the earth's atmosphere may be conveniently referred to in the following works:

1. Basset's Treatise on Hydrodynamics, Vol. II, pp. 34–94, 1888.
2. Lamb's Hydrodynamics, pp. 222–265, 1895.
3. Wien's Lehrbuch der Hydrodynamik, pp. 54–83, 1900.
4. Bigelow's Summary of Formulas, Cloud Report, pp. 508–513, 1898.

Since the notation differs in these treatises the following table of equivalents is added:

TABLE 1.—*Equivalent systems of notation.*

Functions.	Basset.	Lamb.	Wien.	Bigelow.
Total differential	∂	∂	d	d
Partial differential	d	d	∂	∂
Differential increment	δ	δ	δ	δ
Finite difference	Δ	Δ	Δ	Δ
Rectangular coordinates	y, z, x	y, z, x	x, y, z	x, y, z
Cylindrical coordinates	ϖ, θ, z	ϖ, ϑ, x	ρ, ϑ, z	ϖ, ϕ, z
Polar coordinates	r, θ, ϕ	r, θ, ω	r, ϑ, Θ	r, θ, λ
Rectangular velocity	v, w, u	v, w, u	u, v, w	u, v, w
Cylindrical velocity	v, w, u	v, w, u	r, ϖ, η, w	u_1, v_1, w_1
Polar velocity	V, W, U	v, w, u	....	u_2, v_2, w_2
Angular velocity	η, ζ, ξ	η, ζ, ξ	ξ, η, ζ	$\omega_1, \omega_2, \omega_3$
Current function	Right hand. $+\psi$	Right hand. $-\psi$	Left hand. $+\psi$	Right hand. $+\psi$
Velocity potential	$+\phi$	$-\phi$	$+\phi$	$+\phi$
Static potential	$+V$	$+V$	$-V$	$+V$
Vortical coordinates	M, N, L	G, H, F	U, V, W	F, G, H
Kinetic energy	T	T	L	T
Potential energy	V	V	F	U
Density	ρ	ρ	s	ρ
Viscosity coefficient	$\nu = \frac{\mu}{\rho}$	μ	k^2	μ

Bigelow and Wien take the z-axis as the axis of rotation in cylindrical coordinates, while Basset and Lamb use the x-axis. Wien has left-hand rotation and the others right-hand.

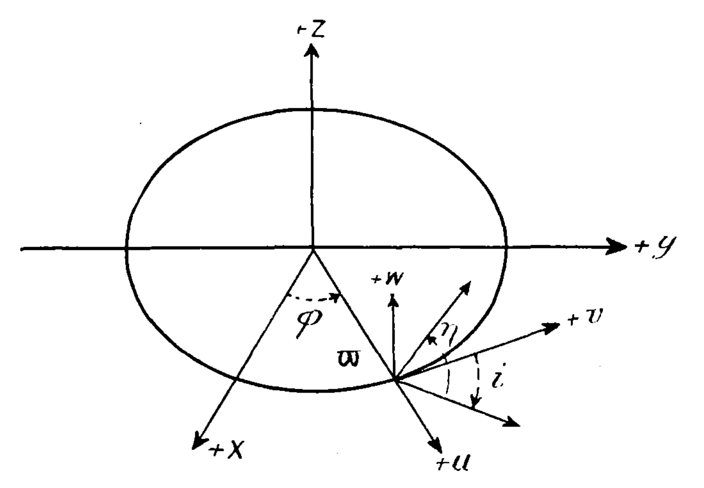

FIG. 1.—Rectangular coordinates of any point are x, y, z. Cylindrical coordinates of the same point are ϖ, ϕ, z. Velocities at that point (ϖ, ϕ) are u, v, w. Angles at that point (ϖ, ϕ) are i, η.

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[1] See Monthly Weather Review for July, 1906, p. 307–315, and plates.

[2] Report of the Chief of the Weather Bureau, 1898–99, Vol. II. Hereafter this is referred to as "Cloud Report," or merely "C. R."

[3] Vol. XXXII, p. 15–20.

TABLE 2.—*Equations in cylindrical coordinates.*

(Compare equations 152, 160, 161, 162, 163, 165......pp. 497, 499, 500, Cloud Report.)

(1) Linear displacements. Cloud Report 152.

$$\begin{cases}\partial x = u\partial t = \partial\varpi. & x = \varpi\cos\varphi.\\ \partial y = v\partial t = \varpi\partial\varphi. & y = \varpi\sin\varphi.\\ \partial z = w\partial t = \partial z.\end{cases}$$

(2) Angular velocities and forces in symmetrical motion about the z-axis. Cloud Report 160.

$$\begin{cases}\omega_1 = 0.\\ \omega_2 = 0.\\ \omega_3 = +\dfrac{v}{\varpi}.\end{cases}\qquad \begin{cases}-\dfrac{\partial V}{\partial x} = -\dfrac{\partial V}{\partial\varpi} = 0.\\ -\dfrac{\partial V}{\partial y} = -\dfrac{\partial V}{\varpi\partial\varphi} = 0.\\ -\dfrac{\partial V}{\partial z} = -g.\end{cases}$$

(3) Linear velocities. Cloud Report 152.

$$\begin{cases}u = \dfrac{dx}{dt} = u_1\cos\varphi - v_1\sin\varphi.\\ v = \dfrac{dy}{dt} = u_1\sin\varphi + v_1\cos\varphi.\end{cases}$$

(4) Linear velocities with moving axes in cylindrical coordinates. Cloud Report 160.

$$\begin{cases}u_1 = \dfrac{\partial\varpi}{\partial t} - y\omega_3 + z\omega_2.\\ v_1 = \varpi\dfrac{\partial\varphi}{\partial t} - z\omega_1 + x\omega_3.\\ w_1 = \dfrac{\partial z}{\partial t} - x\omega_2 + y\omega_1.\end{cases}$$

(5) Angular velocities omitting the subscripts in u_1, v_1, w_1. Cloud Report 162.

$$\begin{cases}2\omega_1 = \dfrac{\partial w}{\varpi\partial\varphi} - \dfrac{\partial v}{\partial z}.\\ 2\omega_2 = \dfrac{\partial u}{\partial z} - \dfrac{\partial w}{\partial\varpi}.\\ 2\omega_3 = \dfrac{\partial v}{\partial\varpi} - \dfrac{\partial u}{\varpi\partial\varphi} + \dfrac{v}{\varpi}.\end{cases}$$

(6) General equations of motion symmetrically about the z-axis. Cloud Report 161.

$$\begin{cases}-\dfrac{\partial V}{\partial\varpi} - \dfrac{\partial P}{\rho\partial\varpi} = \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial\varpi} + w\dfrac{\partial u}{\partial z} - \dfrac{v^2}{\varpi}.\\ 0 = \dfrac{\partial v}{\partial t} + u\dfrac{\partial v}{\partial\varpi} + w\dfrac{\partial v}{\partial z} + \dfrac{uv}{\varpi}.\\ -\dfrac{\partial V}{\partial z} - \dfrac{\partial P}{\rho\partial z} = \dfrac{\partial w}{\partial t} + u\dfrac{\partial w}{\partial\varpi} + w\dfrac{\partial w}{\partial z}.\end{cases}$$

(7) Equation of continuity. Cloud Report 165.

$$\frac{\partial(\varpi u)}{\partial\varpi} + \frac{\partial V}{\partial\varphi} + \varpi\frac{\partial w}{\partial z} = \frac{\partial u}{\partial\varpi} + \frac{u}{\varpi} + \frac{\partial w}{\partial z} = 0.$$

(8) General equations of motion on the rotary earth. Cloud Report 165.

$$\begin{cases}-\dfrac{\partial P}{\rho\partial\varpi} = \dfrac{du}{dt} - 2n\cos\theta\cdot v - \dfrac{v^2}{\varpi} + ku.\\ -\dfrac{\partial P}{\rho\varpi\partial\varphi} = \dfrac{dv}{dt} + 2n\cos\theta\cdot u + \dfrac{uv}{\varpi} + kv.\\ -\dfrac{\partial P}{\rho\partial z} = \dfrac{dw}{dt} + g.\end{cases}$$

(9) It is convenient usually to take the positive direction of the z-axis upward, but to place the plane $x\,y$ below the sea-level surface.

The velocity coordinates u, v, w in forms of the current function ψ.

In discussing problems in vortex motion, it is convenient to use the current function ψ, which is deduced from the equation of continuity. This equation is:

$$(10)\qquad \frac{\partial u}{\partial\varpi} + \frac{u}{\varpi} + \frac{\partial w}{\partial z} = 0,$$

and it may take the form,

$$(11)\qquad \frac{1}{\varpi}\frac{\partial}{\partial\varpi}(u\varpi) + \frac{\partial w}{\partial z} = 0.$$

This is satisfied by substituting the velocities,

$$(12)\qquad u = -\frac{1}{\varpi}\frac{\partial\psi}{\partial z},\quad w = +\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi},$$

which are known as Stokes's functions.

In order to satisfy the second equation of motion in 161, where the motion is steady and $\frac{\partial v}{\partial t} = 0$, the value of v is,

$$(13)\qquad v = \frac{\psi}{\varpi},$$

so that $\psi = v\varpi$ is the constant in vortex motion.

Substituting these values of u, v, w in $(161)_2$, we have,

$$(14)\qquad -\frac{1}{\varpi^2}\frac{\partial\psi}{\partial\varpi}\frac{\partial\psi}{\partial z} + \frac{1}{\varpi}\frac{\partial\psi}{\partial z}\frac{\psi}{\varpi^2} + \frac{1}{\varpi^2}\frac{\partial\psi}{\partial z}\frac{\partial\psi}{\partial\varpi} - \frac{1}{\varpi}\frac{\partial\psi}{\partial z}\frac{\psi}{\varpi^2} = 0.$$

In the case of steady motion, if the second equation of 161 is multiplied by ϖ, we have,

$$(15)\qquad uv + u\varpi\frac{\partial v}{\partial\varpi} + w\varpi\frac{\partial v}{\partial z} + wv\frac{\partial\varpi}{\partial z} = 0.$$

Since $\frac{\partial \varpi}{\partial z} = 0$, it may also be written,

$$(16) \qquad u \frac{\partial}{\varpi}(\varpi v) + w \frac{\partial}{\partial z}(\varpi v) = 0.$$

This shows that $\psi = v\varpi =$ constant is a solution and develops the vortex law required. Any function of ψ which satisfies this equation will be a solution of the second equation of motion.

(17) Hence, $\varpi v = f(\psi) =$ an arbitrary function of ψ, is a solution of the second equation of motion.

We can eliminate the potential and pressure terms from the first and third equations of motion by differentiating the first to z, the third to ϖ, subtracting and substituting the angular velocity, $2\omega_2 = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial \varpi}$, in terms of ψ. Following these precepts we obtain the general vortex equation.

$$(18) \qquad 0 = \frac{\partial}{\partial t}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial \varpi}\right) + \frac{\partial u}{\partial \varpi}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial \varpi}\right) + \frac{\partial w}{\partial z}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial \varpi}\right) - \frac{\partial}{\partial z}\left(\frac{v^2}{\varpi}\right).$$

The following auxiliaries are found from u and ϖ,

$$(19) \qquad \frac{\partial u}{\partial \varpi} = - \frac{\partial}{\partial \varpi}\frac{1}{\varpi}\frac{\partial \psi}{\partial z}. \qquad v\varpi = f(\psi).$$

$$(20) \qquad \frac{\partial w}{\partial z} = + \frac{\partial}{\partial z}\frac{1}{w}\frac{\partial \psi}{\partial \varpi}. \qquad \frac{v^2}{\varpi} = \frac{[f(\psi)]^2}{\varpi^3}$$

$$(21) \qquad 2\,\omega_2 = \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial \varpi}\right) = - \frac{1}{\varpi}\left(\frac{\partial^2 \psi}{\partial \varpi^2} - \frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi} + \frac{\partial^2 \psi}{\partial z^2}\right)$$

Making the substitutions, we obtain

$$(22) \quad 0 = \frac{\partial}{\partial t}(2\,\omega_2) + \frac{\partial u}{\partial \varpi}(2\,\omega_2) + \frac{\partial w}{\partial z}(2\,\omega_2) - \frac{\partial}{\partial z}\frac{[f(\psi)]^2}{\varpi^3}$$

Hence,

$$(23) \qquad 0 = \frac{1}{\varpi}\frac{\partial}{\partial t}\left(\frac{\partial^2 \psi}{\partial \varpi^2} - \frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi} + \frac{\partial^2 \psi}{\partial z^2}\right) + \frac{\partial \psi}{\partial z}\cdot\frac{\partial}{\partial \varpi}\left[\frac{1}{\varpi^2}\left(\frac{\partial^2 \psi}{\partial \varpi^2} - \frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi} + \frac{\partial^2 \psi}{\partial z^2}\right)\right]$$
$$- \frac{\partial \psi}{\partial \varpi}\cdot\frac{\partial}{\partial z}\left[\frac{1}{\varpi^2}\left(\frac{\partial^2 \psi}{\partial \varpi^2} - \frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi} + \frac{\partial^2 \psi}{\partial z^2}\right)\right] - \frac{2 f(\psi)\frac{\partial f(\psi)}{\partial z}}{\varpi^3}.$$

Any function of ψ satisfying this equation is capable of giving a vortex motion. In the application to the atmospheres of the earth and the sun some simple forms will be considered and illustrated by examples from the observations. Inasmuch as it is not possible to make observations in all parts of the tornadoes, hurricanes, and cyclones, it has been very difficult to secure the values of the constants entering into the formulas, but it is thought that this trouble has now been overcome. The simultaneous operation of the current function ψ in equations (17) and (23) is necessary in order to combine the velocities u, v, w in a consistent vortical motion. That we may make it clear in what respects the solutions adopted in these papers differ from other solutions found in previous discussions, the following brief recapitulation is summarized from my Cloud Report, 1898–99, p. 595–603.

Ferrel's solution.—Ferrel took the second equation of (8), and for assumed symmetry about the z-axis with no friction, $kv=0$, reduced it to the form,

$$(24) \qquad \frac{dv}{dt} + (2n\cos\theta + \nu_1)\,u = 0.$$

From this by integration within a fixt cylindrical surface of radius ϖ_0, he deduced the tangential velocity

$$(25) \qquad v = \left(\frac{\varpi_0^2}{2\,\varpi^2} - 1\right)\varpi\, n \cos\theta,$$

at the distance ϖ from the axis. The angle of divergence of the stream line on the horizontal plan from the tangent to the isobar, in terms of the coefficient of the deflecting force, $\lambda = 2\,n\cos\theta$, and the coefficient of friction k, is deduced to be, for $\frac{du}{dt} = 0$,

$$(26) \qquad \tan i = \frac{k}{\lambda + \frac{v}{\varpi}}$$

This signifies that the cause of the departure of the currents entering the closed isobars is the effect of the friction and the deflecting force upon the tangential component. But we shall show that the angle i is due to an entirely different set of circumstances as a primary cause, tho its normal value on a given level or stratum may be slightly modified by these two auxiliary forces.

The German solution.—The second equation of motion has generally been discust in a different manner by the German meteorologists, who have used two other solutions of which it is capable when the fuller form is employed, namely,

$$(27) \qquad \frac{\partial v}{\partial t} + \frac{uv}{\varpi} + \lambda u + kv = 0.$$

These two-type solutions are common to the works of Guldberg and Mohn, Sprung, Oberbeck, Pockels, and others, wherein Oberbeck and Pockels have introduced modifying factors into the simple solution of Guldberg and Mohn or Sprung. One solution is taken applicable to the inner part of a cyclone, and the other to the outer part.

		First solution (inner part).	Second solution (outer part).
(28)	Radial velocity	$u = -\frac{c}{2}\varpi.$	$u = -\frac{c}{\varpi}$
(29)	Tangential velocity	$v = +\frac{\lambda}{k-c}\cdot\frac{c}{2}\varpi z.$	$v = +\frac{\lambda}{k}\frac{c}{\varpi}z$
(30)	Vertical velocity	$w = +cz.$	$w = 0$
(31)	Angle of inclination,	$\tan i = \frac{u}{v} = -\frac{k-c}{\lambda z}.$	$\tan i = \frac{u}{v} = -\frac{k}{\lambda z}$
(32)	Current function $(u.w)$	$\psi_1 = \frac{c}{2}\varpi^2 z.$	$\psi_1 = cz$
	Current function (v)	$\psi_2 = \frac{c}{2}\frac{\lambda}{k-c}\varpi^2 z.$	$\psi_2 = c\frac{\lambda}{k}z$

It is seen that these solutions depend upon three constants: k, the coefficient of friction; λ, the coefficient of the deflecting force due to the earth's rotation; and c, the coefficient of the vertical distance z from the plane of reference, in order to produce the observed angle of inclination. The current functions derived from these solutions are, however, inconsistent. If Stokes's functions be applied to u and w in the first solution, then $\psi_1 = \frac{c}{2}\varpi^2 z$; but if the vortex law, $\psi = v\varpi =$ constant, is used, then $\psi_2 = \frac{c}{2}\frac{\lambda}{k-c}\varpi^2 z$, which is a different value of ψ. In the same way, by means of Stokes's functions, $(u\ w)$ give for the current function $\psi_1 = cz$, while the vortex law $\psi_2 = v\varpi = c\frac{\lambda}{k}z =$ a constant, but differing in value from ψ_1. Hence, $\psi_2 =$ const. ψ_1; or $\psi_2 =$ const. ψ_1. In nature, there is no outer part of a cyclone where $w = 0$, and there is no boundary where the law of motion changes suddenly from the parabolic type, $\frac{v}{\varpi} =$ constant, to the hyperbolic type, $v\varpi =$ constant, as is called for in these solutions. Nor is it possible that the natural values of k, λ, c can account for the observed angle i in all levels, and they are by no means constant even on the same vortex tube.

SOLUTION FOR THE FUNNEL-SHAPED VORTEX TUBE. COTTAGE CITY WATERSPOUT, CHAMBERLAIN 3d A.

Since my solution for the vortex represented in Chamberlain's photograph 3d A of the Cottage City waterspout, MONTHLY WEATHER REVIEW, July, 1906, Plate VIII, approximates the type which is involved in the first solution, inner part, as applied by the German meteorologists to the cyclone, I will take up that problem before the others, and will then illustrate the other type by Chamberlain's 2d A, Plate I, the St. Louis tornado, and the De Witte typhoon. The ocean cyclone and the land cyclone are impure vortices of the latter type. Unfortunately, by adopting the present procedure it is not possible at the outset to demonstrate my method of finding the values of the constants required in the evaluation of the formulas in this special case. Having only the photograph of the tube, which gives the outline of the vortex, but no idea of the velocities in the several directions, it has been exceedingly difficult to discover what the vortex constants are in nature. They were finally obtained by starting with the hurricane, and advancing thru the tornado and the second waterspout vortex to the first type now under consideration. Many efforts were made before this successful result was obtained, the outcome being now checked by reproducing a vortex whose dimensions agree closely with that represented in the photograph when the latter is translated into meters by the scale already found to apply, namely, *1 millimeter on the photograph = 18.3 meters at the waterspout.* The derivation of the formulas is very simple after the form of the vortex function has been determined. That form which is applicable to the funnel-shaped vortex tube is,

$$\psi = C\varpi^2 z. \tag{33}$$

From this formula we find by differentiation,

$$\frac{\partial \psi}{\partial \varpi} = 2C\varpi z, \quad \frac{\partial^2 \psi}{\partial \varpi^2} = 2Cz, \text{ and,} \tag{34}$$

$$\frac{\partial \psi}{\partial z} = C\varpi^2, \quad \frac{\partial^2 \varpi}{\partial z^2} = 0, \quad \text{so that,} \tag{35}$$

$$\frac{\partial^2 \psi}{\partial \varpi^2} - \frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi} + \frac{\partial^2 \psi}{\partial z^2} = 2Cz - 2Cz + 0 = 0, \text{ and hence} \tag{36}$$

the general vortex equation (23) is satisfied. The last term is obtained from the centrifugal force. Thus, since $f(\psi) = v\varpi = C\varpi^2 z$, we have,

$$\frac{2f(\psi)\dfrac{\partial f(\psi)}{\partial z}}{\varpi^3} = \frac{2C\varpi^2 z \,.\, C\varpi^2}{\varpi^3} = 2C^2\varpi z. \tag{37}$$

The structure of the vortex is such that the following relations hold true on the same level, as will be illustrated in the discussion of the Cottage City waterspout.

(38)	Ratio.	$\log \rho = \log \frac{\varpi_n}{\varpi_{n+1}}$	= the ratio between successive tubes.
(39)	Constant.	$\log C_n = \log C_0 + 2n \log \rho$	$= \log C_0 + 2n \log \frac{\varpi_n}{\varpi_{n+1}}$.
(40)	Radius.	$\log \varpi_n = \log \varpi_0 - n \log \rho$	$= \log \varpi_0 - n \log \frac{\varpi_n}{\varpi_{n+1}}$.
(41)	Radial.	$\log u_n = \log u_0 + n \log \rho$	$= \log u_0 + n \log \frac{\varpi_n}{\varpi_{n+1}}$.
(42)	Tangential.	$\log v_n = \log v_0 + n \log \rho$	$= \log v_0 + n \log \frac{\varpi_n}{\varpi_{n+1}}$.
(43)	Vertical.	$\log w_n = \log w_0 + 2n \log \rho$	$= \log w_0 + 2n \log \frac{\varpi_n}{\varpi_{n+1}}$.
(44)	Horizontal.	$\log \tan i_n =$ constant.	
(45)	Vertical.	$\log \tan \eta_n = \log \tan \eta_0 + n \log \rho$	$= \log \tan \eta_0 + n \log \frac{\varpi_n}{\varpi_{n+1}}$.
(46)	Time.	$\log t_n = \log t_0 - 2n \log \rho$	$= \log t_0 - 2n \log \frac{\varpi_n}{\varpi_{n+1}}$.
(47)	Volume.	$\text{Volume} = \pi \left(\varpi_n^2 - \varpi_{n+1}^2\right) w_m$	= constant.
(48)	Centrifugal.	$\log \left(\frac{v^2}{\varpi}\right)_n = \log \left(\frac{v^2}{\varpi}\right)_0 + 3n \log \rho$	$= \log \left(\frac{v^2}{\varpi}\right)_0 + 3n \log \frac{\varpi_n}{\varpi_{n+1}}$.
(49)	Pressure.	$\log \frac{B_n - B_{n+1}}{B_{n-1} - B_n} - \log \frac{\varpi_n}{\varpi_{n+1}}$	$= \log \rho = \log \frac{\varpi_n}{\varpi_{n+1}}$.

Formulas for the radial, tangential, and vertical velocities.

It will be convenient to use different coordinate axes in solving the two types of vortices represented respectively by the funnel-shaped and the dumb-bell-shaped tubes, photographed in Chamberlain 3d A and 2d A. For the former the z-axis should be taken *positive downward* from a reference plane near the base of the cumulus cloud from which the vortex is projected; for the latter the reference plane is below the surface of the sea, and the z-axis is *positive upward.* The reason for this change in the direction of the coordinates will clearly appear in the discussion of the examples of the dumb-bell vortex.

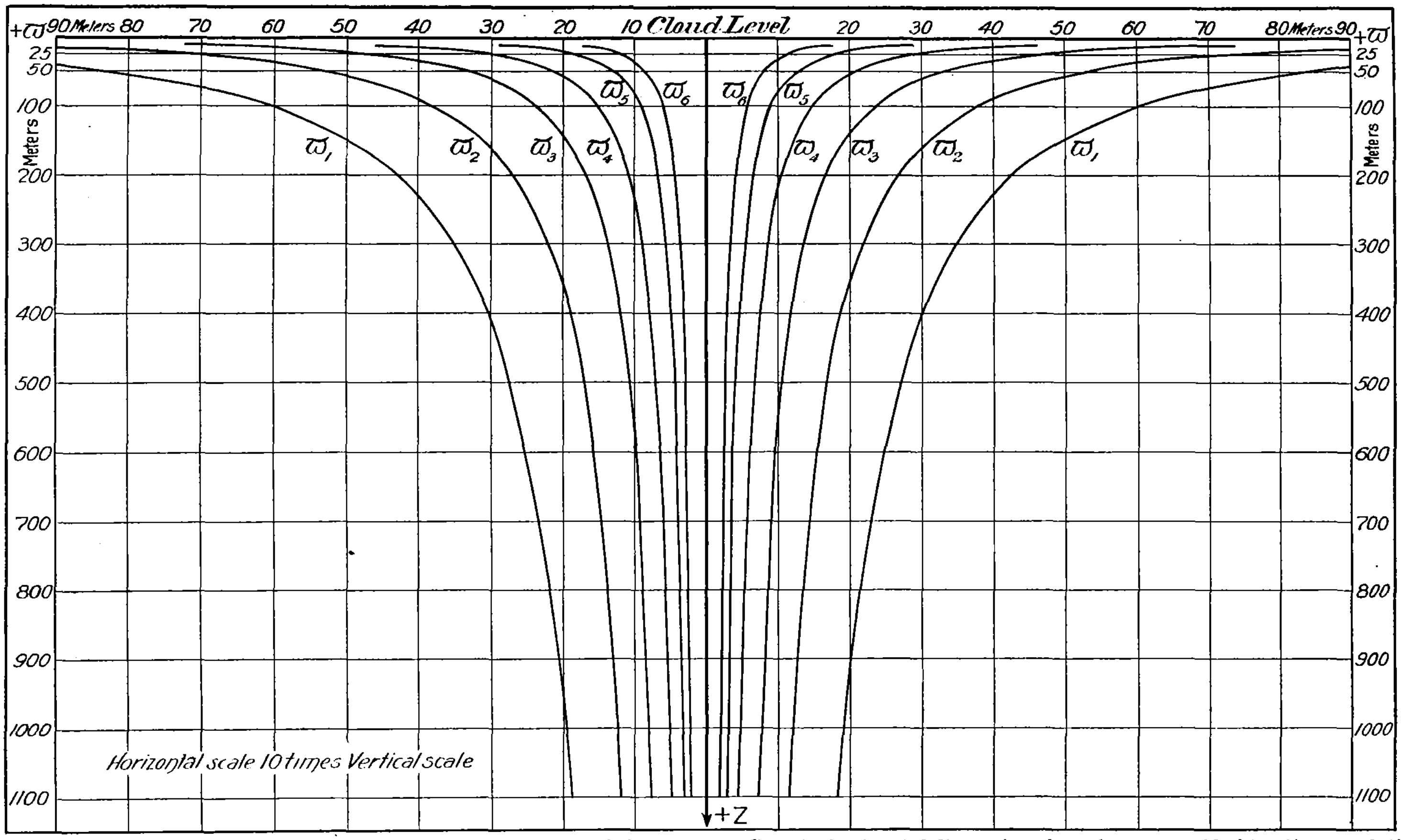

FIG. 2.—The (ϖz) lines in a funnel vortex for different values of the constant C. The horizontal dimensions have been magnified ten times, relative to the vertical dimensions, in order to exhibit the internal structure.

TABLE 3.—*Formulas applicable to the funnel vortex.*

Number of column.	1	2	3	4	5	6
Current function.	$\psi =$	$\varphi z =$	$C\varpi^2 z =$	$v\varpi =$	$u\varpi z =$	$-\frac{w}{2}\varpi^2.$
Constant of vortices.	$C =$	$\frac{\varphi}{\omega^2} =$	$\frac{\psi}{\varpi^2 z} =$	$\frac{v}{\varpi z} =$	$\frac{u}{\varpi} =$	$-\frac{w}{2z}.$
Radial velocity.	$u =$	$\frac{1}{\varpi}\frac{\partial\psi}{\partial z} =$	$C\varpi =$	$\frac{v}{z} =$	$\frac{\psi}{\varpi z} =$	$-\frac{w\varpi}{2z}.$
Tangential velocity.	$v =$	$\frac{\psi}{\varpi} =$	$C\varpi z =$	$uz =$	$\frac{\psi}{\varpi} =$	$-\frac{w}{2}\varpi.$
Vertical velocity.	$w =$	$-\frac{1}{\varpi}\frac{\partial\psi}{\partial\varpi} =$	$-2Cz =$	$-\frac{2uz}{\varpi} =$	$-\frac{2\psi}{\varpi^2} =$	$-\frac{2v}{\varpi}.$

The formulas for the radial (u), tangential (v), and vertical (w) velocities are given in Table 3, together with several check combinations.

Having adopted the form of current function, ψ, then the radial and vertical velocities are found from Stokes's functions and the tangential velocity from the vortex law, $\varphi = v\varpi =$ constant. It is evident that one value of the constant C holds true for a single stream line (ϖz), but changes its value from one vortex tube to another. Thus, for the lines in the Cottage City waterspout, we have for—

Line (1) $C_1 = 0.001111.$
Line (2) $C_2 = 0.002862.$
Line (3) $C_3 = 0.007372.$
Line (4) $C_4 = 0.018990.$
Line (5) $C_5 = 0.048910.$
Line (6) $C_6 = 0.126000.$

It was for a long while impossible to discover a method for computing these values of C_1, C_2, etc., because no velocities but only the dimensions of the outer sheath (1) were available for use. It is seen by the formulas that the dimensions of the vortex depend upon C, even when the (ϖ, z) are known, so that if the height z and the radius ϖ are given at successive points it is yet necessary to know C before the velocities can be computed even approximately. The velocities (u, v, w) all increase with C, and hence they are all greater in the interior in proportion to the approach to the axis; u increases but v and w diminish with approach to the plane of reference at the base of the cloud, as determined by the formulas in column 3.

In a vortex of this kind the simplest relation is that the ratios of the successive radii are equal and constant, so that,

$$\rho = \frac{\varpi_1}{\varpi_2} = \frac{\varpi_2}{\varpi_3} = \frac{\varpi_3}{\varpi_4} = \ldots \frac{\varpi_n}{\varpi_{n+1}}$$

If the values of the radii of successive isobars can be measured, ϖ_1 (outer), ϖ_2, ϖ_3, ... ϖ_n (inner) the value of ρ can be readily computed. The approximate radii of the circular isobars in hurricanes and cyclones can be thus measured on the charts, and from these the value of ρ, and log ρ, determined as a mean. Thus for four cases, i. e., the De Witte hurricane, the St. Louis tornado, a typical large ocean cyclone, and a typical large land cyclone, I have computed log ρ from the available data. (See Table 4.)

TABLE 4.—*Values of log ρ in several vortices.*

De W. hurricane.			S. L. tornado.			Ocean cyclone.			Land cyclone.		
B	*log ρ*		*B*	*log ρ*		*B*	*log ρ*		*B*	*log ρ*	
760			755			755			760		
	0.20412	Constant.		0.20412	Constant.		0.10266	Constant 0.21000 (10 mm.)		0.10791	Variable.
750			745			750			755		
	0.19269			0.20412			0.10914			0.12390	
740			735			745			750		
	0.21904			0.19382			0.10003			0.15924	
730			725			740			745		
	0.16230			0.23408			0.10959			0.22874	
720			715			735			740		
	0.23798			0.19189			0.10400			0.43573	
710			705			730			735		
	0.22578			0.21388			0.12665	Variable.			
700			695			725					
	0.19626			0.19629			0.16428				
690			685			720					
							0.24055				
Means ...	0.20563			0.20546			0.10500				
	1.6056			1.60493			1.27350				

In the hurricane and tornado, log ρ is practically constant and nearly the same in value; in the ocean cyclone it is constant outside of the isobar 730, but increases in value toward the axis from isobar 730 to isobar 715, showing that the ocean cyclone is not a pure vortex near the center. In the land cyclone, log ρ is not constant, but enlarges in the same ratio that occurs near the center of the ocean cyclone, showing that the land cyclones do not follow the pure vortex law, even approximately.

Since the Cottage City waterspout resembles the pure vortices of the tornado and hurricane more than the imperfect vortices of the ocean and land cyclones, it is proper to adopt log $\rho = 0.20546$ as an approximate value. It may be found that some such value of log ρ is a characteristic of the earth's atmosphere, when its small vortices develop freely; that is, it may be a typical constant, while other atmospheres may operate according to a different constant.

The current function constant,

$$\log \psi = \log (v\,\varpi) = 2.60206,$$

has been determined by a series of trials, which it is not necessary here to enumerate. If it were possible to measure the tangential velocity v at any point $(\varpi_1 z)$ in the vortex, as, for instance, on the sheath, where it begins to expand rapidly before merging with the cloud, then we should have $\psi = v\,\varpi =$ constant. Several such measures at different points on the sheath $(v_1\,\varpi_1)$, $(v_2\,\varpi_2)$, etc., would give several values for the constant, and the mean could be taken as available thruout the vortex. This can be done for the tornado and the hurricane on the ground, or at the sea level; but with the waterspout it is possible only to assume certain values of v at a given height, z, measure ϖ, compute the tube from the cloud to the sea level, and by interpolation compare with the observed dimensions as taken from the photograph. It was finally determined to adopt the following initial data:

$$\text{At height } z = 100 \text{ meters.} \left\{ \begin{array}{l} \varpi = 60 \text{ meters.} \\ v = 6.67 \text{ meters per second.} \\ \log \psi = 2.60206 \\ \log \rho = 0.20546 \end{array} \right.$$

Table 5 shows the manner in which the tube obtained from the computation to be given matches the dimensions scaled from the photograph, Chamberlain 3d A.

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TABLE 5.—*Comparison of the computed and observed radii, Chamberlain 3d A, in meters.*

Height z.	Radius computed ϖ	Radius Chamberlain 3d A.*
0	∞	....
1	600.0	(600)
2	424.3	....
5	268.4	....
10	189.7	200
25	120.0	125
50	84.9	85
100	60.0	60
200	42.4	43
300	34.6	35
400	30.0	30
500	26.8	25
600	24.5	23
700	22.7	22
800	21.2	20
900	20.0	19
1000	19.0	18
1100	18.1	....

*1 millimeter on photograph = 18.3 meters at the waterspout.

There is some uncertainty in tracing the form of the vortex head near the cloud, but the darkening of the cloud in Chamberlain 3d A and 3d B indicates that the vortex spreads out to about 1200 meters in diameter, something like the height of the cloud base from the sea level. This gives 600 meters radius near the plane of reference, as in the table. At the bottom the tube is surrounded by a lofty cascade, which prevents the measurement of the radius at the level $z = 1100$ meters.

The constants C are found at first from computations with log ψ, ϖ, z on the level $z = 100$, using the measured radius ϖ_1, and applying log ρ to the log ϖ_1 by formula (30), in succession from the outer to the inner tubes, which are supposed to be separated from each other by the pressure in millimeters of mercury, as determined by (49). An example of the preliminary computation is shown in full in Table 6.

The results of Table 6, Section I, there computed for the height z=100 meters, are entered in sections I, II, III, table 7, in the appropriate line, and printed in heavier type. The other parts of these tables are to be computed from these data for all the other altitudes. Having computed the radial distance from the axis at all altitudes in order to find the *radial* component u, it is only necessary to multiply ϖ by the C of the respective lines; to find the *tangential* component it is enough to multiply u at each point by the height z; and to obtain the *vertical* component it is sufficient to multiply $-2C$ by the height z. In this vortex the component velocities and the coordinate distances stand in very simple relations, and this is probably one reason why the atmosphere tends to circulate according to this simple solution of the second equation of motion.

An inspection of Table 7, Sections I, II, III, shows that the following facts hold true in regard to the velocities. The *radial* component u increases slowly upward thru the long, tapering tube till very near the cloud base, and it then increases very rapidly; it is greater in the interior of the vortex than in the outside tubes, showing that the inner helices slope outward more rapidly than do the outer ones; it is probable that the extreme actual radial velocity in a horizontal plane near the cloud is practically about 5 meters per second where the tangential rotating velocity disappears. The *tangential* velocity decreases rapidly upward, especially in the inner tubes, and it increases rapidly from the outer tube toward the axis, where it may amount to 200 meters per second, or 447 miles per hour. It is not probable that such enormous velocities exist in the atmosphere even under vortex conditions, but a pure vortex evidently develops tremendous gyratory motions very near the axis. The *vertical* velocity decreases rapidly upward, more so as the tubes diminish their dimensions; but it increases toward the axis, where it may attain the enormous velocity of 250 meters per second, or 559 miles per hour. In the extreme total velocity, at the point where this computation ends, if the vortex actually develops so near the axis, we have,

$$q_5 = [(0.13)^2 + (146.7)^2 + (107.6)^2]^{1/2} = 182 \text{ meters per second, or 407 miles per hour.}$$

In Table 8 are given the total velocity q at numerous points within the vortex, the horizontal angle i, which it makes in the plane at the height z, and at the point ϖ, φ with the tangent to the circle; also the vertical angle η, which it makes at the same point with the tangent. (See fig. 1.) The angle i is positive outward and the angle η is negative upward in this system of coordinates. Section I of this table shows that the angle i is the same for each tube on a given section at the height z, and it increases upward slowly thru the long, tapering tube and very rapidly in the last 10 meters, where the motion of q is becoming asymptotic to the plane of reference. The angle η decreases upward and becomes zero at the cloud base; it increases rapidly from the outer tube toward the axis, and seems to be limited by the angle 36° on tube 5. The angle of the pitch of the helix is steeper near the center of the

TABLE 6.—*Cottage City Waterspout. Chamberlain, 3d A. Computation of the radius ϖ thruout the vortex.*

Assume $\varpi = 60$ at $z = 100$; $\log \psi = 2.60206$; $\log \rho = 0.20546$.

I. Line.	(1)	(2)	(3)	(4)	(5)	(6)	Formula.
$\log \varpi$	1.77815	1.57269	1.36723	1.16177	0.95631	0.75085	
ϖ	60.0	37.4	23.3	14.5	9.0	5.6	$\varpi_{n+1} = \frac{\varpi_n}{\rho}$.
$\log \varpi^2$	3.55630	3.14538	2.73446	2.32354	1.91262	1.50170	
z	2.00000	2.00000	2.00000	2.00000	2.00000	2.00000	
$\varpi^2 z$	5.55630	5.14538	4.73446	4.32354	3.91262	3.50170	
$\log C$	7.04576	7.45668	7.86760	8.27852	8.68944	9.10036	
C	0.001111	0.002862	0.007372	0.01899	0.04891	0.12600	$C = \frac{\psi}{\varpi^2 z}$.
$\log C\varpi$	8.82391	9.02937	9.23483	9.44029	9.64575	9.85121	
u	0.06667	0.1070	0.1717	0.2756	0.4423	0.7099	$u = C\varpi$.
$\log \frac{\psi}{\varpi}$	0.82391	1.02937	1.23483	1.44029	1.64575	1.85121	
v	6.67	10.70	17.17	27.56	44.23	70.99	$v = \frac{\psi}{\varpi}$.
$-\log 2\,Cz$	—9.34679	—9.75771	—0.16863	—0.57955	—0.99047	— 1.40139	
w	—0.222	—0.572	—1.474	—3.798	—9.783	—25.199	$w = -2\,Cz$.

These data can be used to test the other formulas given in Table 3. In computing from the 100-meter level to other values of (ϖz), we proceed as follows, showing as examples a few of the levels only for $C_1 = 0.00111$; $\varpi^2 = \frac{\psi}{C_1 z}$:

Radius ϖ in all parts of the vortex.

II. z	$\log z$	$\log C_1 z$	$\log \varpi^2$	$\log \varpi_1$	$\log \varpi_2$	$\log \varpi$	$\log \varpi_4$	$\log \varpi_5$	$\log \varpi_6$
0	— ∞	— ∞	∞	∞	∞	∞	∞	∞	∞
10	1.00000	8.04576	4.55630	2.27815	2.07269	1.86723	1.66177	1.45631	1.25085
100	2.00000	9.04576	3.55630	**1.77815**	**1.57269**	**1.36723**	**1.16177**	**0.95631**	**0.75085**
200	2.30103	9.34679	3.25527	1.62764	1.42218	1.21672	1.01126	0.80580	0.60034
300	2.47712	9.52288	3.07918	1.53959	1.33413	1.12867	0.92321	0.71775	0.51229
700	2.84510	9.89086	2.71120	1.35560	1.15014	0.94468	0.73922	0.53376	0.32830
1100	3.04139	0.08715	2.51491	1.25746	1.05200	0.84654	0.64108	0.43562	0.23016

III. z				ϖ_1	ϖ_2	ϖ_3	ϖ_4	ϖ_5	ϖ_6
0				∞	∞	∞	∞	∞	∞
1				600.0	373.8	232.9	145.1	90.4	56.3
2				424.3	264.4	164.7	102.6	63.9	39.8
5				268.4	167.2	104.2	64.9	40.4	25.2
10				189.7	118.2	73.7	45.9	28.6	17.8
25	These computations need not be executed.			120.0	74.8	46.6	29.0	18.1	11.3
50				84.9	52.9	32.9	20.5	12.8	8.0
100				60.0	37.4	23.3	14.5	9.0	5.6
200				42.4	26.4	16.5	10.3	6.4	4.0
300				34.6	21.6	13.5	8.4	5.2	3.3
400				30.0	18.7	11.7	7.3	4.5	2.8
500				26.8	16.7	10.4	6.5	4.0	2.5
600				24.5	15.2	9.5	5.9	3.7	2.3
700				22.7	14.1	8.8	5.5	3.4	2.1
800				21.2	13.2	8.2	5.1	3.2	2.0
900				20.0	12.5	7.8	4.8	3.0	1.9
1000				19.0	11.8	7.4	4.6	2.9	1.8
1100				18.1	11.3	7.0	4.4	2.7	1.7

For the other values of C_n following $C_1 = 0.001111$ it is sufficient to subtract $\log \rho = 0.20546$ from the values of $\log \varpi_1$ under C_1 in succession to one another in Section II. The $\log \varpi$ of Section I appears in its place in Section II.

vortex at the sea level than at any other point, the pitch diminishing upward and outward. The total velocity q is greatest near the axis at sea level, it diminishes rapidly outward and upward, and its magnitude near the axis is astonishing.

The time of the rotation of a particle on a given plane is found as follows. The length of the path is $2\pi\varpi$, the velocity v; so that $t = \frac{2\pi\varpi}{v}$. Take as an example the plane $z = 1100$. (See Table 9.)

It takes 5.14 seconds to make one circuit about the axis at the surface of the ocean on the outer tube, and 0.04 second, i. e., one twenty-fifth of a second, on the sixth or inner tube. Subtracting the successive values of log t, $(\log t_1 - \log t_2)\ldots$, the result is $2 \log \rho$ in all cases, so that the time of rotation in the different parts of the vortex can be computed from a few initial values. In this way it is seen that even a few isolated observations of the radius ϖ and velocity v can be used to construct the entire vortex. A single anemometer record in a vortex at a distance ϖ from the center of the track is therefore of great value in theoretical meteorological discussions. A consideration of the forces of pressure involved in these velocities is sufficient to see where the powerful destructive forces arise, whose effects are noted in the débris which mark the track of even a small funnel-shaped tornado tube.

TABLE 7.—*Computation of the radial, tangential, and vertical velocities thruout the vortex.*

I. THE RADIAL COMPONENT, $u = C\varpi$.

z	(1) $\log u_1$	(2) $\log u_2$	(3) $\log u_3$	(4) $\log u_4$	(5) $\log u_5$	(6) $\log u_6$
0	∞	∞	∞	∞	∞	∞
10	9.32491	9.53037	9.73583	9.94129	0.14675	0.35221
100	**8.82391**	**9.02937**	**9.23483**	**9.44029**	**9.64575**	**9.85121**
200	8.67340	8.87886	9.08432	9.28978	9.49524	9.70070
300	8.58535	8.79081	8.99627	9.20173	9.40719	9.61265
700	8.40136	8.60682	8.81228	9.01774	9.22320	9.42866
1100	8.30322	8.50868	8.71414	8.91960	9.12506	9.33052

z	u_1	u_2	u_3	u_4	u_5	u_6
0	∞	∞	∞	∞	∞	∞
1	0.667	1.070	1.717	2.756	4.423	7.099
2	0.471	0.757	1.214	1.949	3.128	5.020
5	0.298	0.479	0.768	1.233	1.978	3.175
10	0.211	0.339	0.544	0.874	1.402	2.250
25	0.133	0.214	0.343	0.551	0.885	1.420
50	0.094	0.151	0.243	0.390	0.626	1.004
100	**0.067**	**0.107**	**0.172**	**0.276**	**0.442**	**0.710**
200	0.047	0.076	0.121	0.195	0.313	0.502
300	0.039	0.062	0.099	0.159	0.255	0.410
400	0.033	0.054	0.086	0.138	0.221	0.355
500	0.030	0.048	0.077	0.123	0.198	0.318
600	0.027	0.044	0.070	0.113	0.180	0.290
700	0.025	0.040	0.065	0.104	0.167	0.268
800	0.023	0.038	0.061	0.097	0.156	0.251
900	0.022	0.036	0.057	0.092	0.147	0.237
1000	0.021	0.034	0.054	0.087	0.140	0.225
1100	0.020	0.032	0.052	0.083	0.133	0.214

II. THE TANGENTIAL COMPONENT, $v = C\varpi z$.

z	(1) $\log v_1$	(2) $\log v_2$	(3) $\log v_3$	(4) $\log v_4$	(5) $\log v_5$	(6) $\log v_6$
0	—∞	—∞	—∞	—∞	—∞	—∞
10	0.32391	0.52937	0.73483	0.94029	1.14575	1.35121
100	**0.82391**	**1.02937**	**1.23483**	**1.44029**	**1.64575**	**1.85121**
200	0.97442	1.17988	1.38534	1.59080	1.79626	2.00172
300	1.06247	1.26793	1.47339	1.67885	1.88431	2.08977
700	1.24646	1.45192	1.65738	1.86284	2.06830	2.27376
1100	1.34460	1.55006	1.75552	1.96098	2.16644	2.37190

z	v_1	v_2	v_3	v_4	v_5	v_6
0	0	0	0	0	0	0
1	0.7	1.1	1.7	2.8	4.4	7.1
2	0.9	1.5	2.4	3.9	6.3	10.0
5	1.5	2.4	3.8	6.2	9.9	15.9
10	2.1	3.4	5.4	8.7	14.0	22.5
25	3.3	5.4	8.6	13.8	22.1	35.5
50	4.7	7.6	12.1	19.5	31.3	50.2
100	**6.7**	**10.7**	**17.2**	**27.6**	**44.2**	**71.0**
200	9.4	15.1	24.3	39.0	62.6	100.4
300	11.6	18.5	29.7	47.7	76.6	122.9
400	13.3	21.4	34.3	55.1	88.5	142.0
500	14.9	23.9	38.4	61.6	98.9	158.7
600	16.3	26.2	42.1	67.5	108.3	173.9
700	17.6	28.3	45.4	72.9	117.0	187.8
800	18.9	30.3	48.6	78.0	125.1	200.8
900	20.0	32.1	51.5	82.7	132.7	213.0
1000	21.1	33.8	54.3	87.2	139.9	224.5
1100	22.1	35.5	57.0	91.4	146.7	235.4

TABLE 7—*Continued.* III. THE VERTICAL COMPONENT, $w = -2Cz$.

z	(1) $\log w_1$	(2) $\log w_2$	(3) $\log w_3$	(4) $\log w_4$	(5) $\log w_5$	(6) $\log w_6$
0	—∞	—∞	—∞	—∞	—∞	—∞
10	8.34679	8.75771	9.16863	9.57955	9.99047	0.40139
100	**9.34679**	**9.75771**	**0.16863**	**0.57955**	**0.99047**	**1.40139**
200	9.64782	0.05874	0.46966	0.88058	1.29150	1.70242
300	9.82391	0.23483	0.64575	1.05667	1.46759	1.87851
700	0.19189	0.60281	1.01373	1.42465	1.83557	2.24649
1100	0.38818	0.79910	1.21002	1.62094	2.03186	2.44278

z	w_1	w_2	w_3	w_4	w_5	w_6
0	0	0	0	0	0	0
1	—0.0022	—0.0057	—0.0147	—0.0380	—0.0978	—0.2520
2	—0.0044	—0.0115	—0.0295	—0.0760	—0.1957	—0.5040
5	—0.0111	—0.0286	—0.0737	—0.1899	—0.4891	—1.260
10	—0.0222	—0.0572	—0.1474	—0.3798	—0.9783	—2.520
25	—0.0556	—0.1431	—0.3686	—0.9495	—2.446	—6.300
50	—0.1110	—0.2862	—0.7372	—1.899	—4.891	—12.60
100	**—0.222**	**—0.572**	**—1.474**	**—3.798**	**—9.783**	**—25.20**
200	—0.444	—1.145	2.949	—7.596	—19.57	—50.40
300	—0.667	—1.717	—4.423	—11.39	—29.35	—75.60
400	—0.889	—2.290	—5.898	—15.19	—39.13	—100.80
500	—1.111	—2.862	—7.372	—18.99	—48.91	—126.00
600	—1.333	—3.434	—8.847	—22.79	—58.70	—151.20
700	—1.556	—4.007	—10.32	—26.59	—68.48	—176.40
800	—1.778	—4.579	—11.79	—30.38	—78.26	—201.60
900	—2.000	—5.152	—13.27	—34.18	—88.05	—226.80
1000	—2.222	—5.724	—14.74	—37.98	—97.83	—252.00
1100	—2.444	—6.297	—16.22	—41.78	—107.61	—277.20

NOTE.—This vortex probably does not develop beyond ϖ_5, u_5 v_5 w_5.

TABLE 8.—*The angles (i, η) which the current having the velocity of q makes with the tangent at (ϖ, ϕ). (Fig. 1.*)*

I.—HORIZONTAL ANGLE i $\left(\tan i = \frac{u}{v}\right)$.

z	(1)	(2)	(3)	(4)	(5)	(6)
0	∞	∞	∞	∞	∞	∞
10	9.00100	9.00100	9.00100	9.00100	9.00100	9.00100
50	8.30104	8.30104	8.30104	8.30104	8.30104	8.30104
100	8.00000	8.00000	8.00000	8.00000	8.00000	8.00000
300	7.52288	7.52288	7.52288	7.52288	7.52288	7.52288
500	7.30104	7.30104	7.30104	7.30104	7.30104	7.30104
700	7.15490	7.15490	7.15490	7.15490	7.15490	7.15490
900	7.04576	7.04576	7.04576	7.04576	7.04576	7.04576
1100	6.95862	6.95862	6.95862	6.95862	6.95862	6.95862

z	(1)	(2)	(3)	(4)	(5)	(6)
	° ′	° ′	° ′	° ′	° ′	° ′
0	90 0	90 0	90 0	90 0	90 0	90 0
10	5 43	5 43	5 43	5 43	5 43	5 43
50	1 9	1 9	1 9	1 9	1 9	1 9
100	0 34	0 34	0 34	0 34	0 34	0 34
300	0 11	0 11	0 11	0 11	0 11	0 11
500	0 7	0 7	0 7	0 7	0 7	0 7
700	0 5	0 5	0 5	0 5	0 5	0 5
900	0 4	0 4	0 4	0 4	0 4	0 4
1100	0 3	0 3	0 3	0 3	0 3	0 3

II.—VERTICAL ANGLE η $\left(-\tan \eta = \frac{w}{v \sec i}\right)$.

z	(1)	(2)	(3)	(4)	(5)	(6)
0	∞	∞	∞	∞	∞	∞
10	8.02071	8.22617	8.43163	8.63709	8.84255	9.04801
50	8.37237	8.57783	8.78329	8.98875	9.19421	9.39967
100	8.52288	8.72834	8.93380	9.13926	9.34472	9.55018
300	8.76144	8.96690	9.17236	9.37782	9.58328	9.78874
500	8.87237	9.07783	9.28329	9.48875	9.69421	9.89967
700	8.94543	9.15089	9.35635	9.56181	9.76727	9.97273
900	9.00000	9.20546	9.41092	9.61638	9.82184	0.02730
1100	9.04358	9.24904	9.45450	9.65996	9.86542	0.07088

z	(1)	(2)	(3)	(4)	(5)	(6)
	° ′	° ′	° ′	° ′	° ′	° ′
0	0 0	0 0	0 0	0 0	0 0	0 0
10	0 36	0 58	1 33	2 29	3 59	6 22
50	1 18	2 10	3 28	5 34	8 53	14 5
100	1 55	3 4	4 54	7 51	12 28	19 33
300	3 18	5 18	8 28	13 25	20 58	31 35
500	4 16	6 49	10 52	17 8	26 19	38 26
700	5 2	8 3	12 48	20 2	30 20	43 12
900	5 43	9 7	14 27	22 28	33 34	46 48
1100	6 19	10 4	15 54	24 34	36 16	49 39

TABLE 8—*Continued.*

III.—TOTAL VELOCITY, $q=(u^2+v^2+w^2)^{\frac{1}{2}}$.

z	(1)	(2)	(3)	(4)	(5)	(6)
0	0.00	0.00	0.00	0.00	0.00	0.00
10	2.12	3.40	5.46	8.77	14.09	22.73
50	4.72	7.57	12.15	19.50	31.30	51.77
100	6.67	10.71	17.18	27.58	44.26	75.34
300	11.6	18.6	30.1	49.1	80.2	144.3
500	14.9	24.1	39.1	64.4	110.3	202.7
700	17.7	28.6	46.6	77.6	135.6	257.7
900	20.1	32.5	53.2	89.4	159.2	311.2
1100	22.3	36.0	59.2	100.5	182.0	363.7

*The vector (q, i, η) makes the angle i on the horizontal plane with the tangent at the point (ϖ, ϕ) and the angle η in the vertical plane at the same point. The sec. i can be neglected except very near the cloud level where the angle i increases rapidly to 90°.

TABLE 9.—*Time to make one circuit at different distances* ϖ.

$z=1100$	(1)	(2)	(3)	(4)	(5)	(6)
log ϖ	1.25746	1.05200	0.84654	0.64108	0.43562	0.23016
log 2π	0.79818					
log $2\pi\varpi$	2.05564	1.85018	1.64472	1.43926	1.23380	1.02834
log v	1.34460	1.55006	1.75552	1.96098	2.16644	2.37190
log t	0.71104	0.30012	9.88920	9.47828	9.06736	8.65644
t	5.14	2.00	0.77	0.30	0.12	0.04

The volume of air transferred upward thru each horizontal section, in the areas bounded by the circles $\varpi_1, \varpi_2, \varpi_3, \varpi_4, \varpi_5, \varpi_6$, at a given elevation, is the same on a given level, and it is also the same thru every horizontal plane. In other words, the volume of air flowing upward is the same in every cylindrical ring-area bounded by the surfaces generated thru the revolution of the lines (ϖ, z) around the central axis. This is easily computed by the formula,

$$\text{volume, } V=\pi\left(\varpi_n^2-\varpi_{n+1}^2\right) w_m,$$

where w_m is the mean vertical velocity on a given ring section. Two examples are taken on the sections for $z=100$, and $z=1100$ meters. Take log ϖ from Table 6, Section II, and log w from Table 7, Section III.

TABLE 10.—*Volume* $=\pi\left(\varpi_n^2-\varpi_{n+1}^2\right) w_m$.

$z=100$	(1)	(2)	(3)	(4)	(5)	(6)	
log ϖ	1.77815	1.57269	1.36723	1.16177	0.95631	0.75085	Table 6, II.
log ϖ^2	3.55630	3.14538	2.73446	2.32354	1.91262	1.50170	
ϖ^2	3600.0	1397.60	542.58	210.64	81.775	31.747	
$\varpi_n^2-\varpi_{n+1}^2$	2202.4	855.02	331.94	128.865	50.028		
log	3.34290	2.93198	2.52106	2.11014	1.69922		
log w	9.34679	9.75771	0.16863	0.57955	0.99047	1.40139	Table 7,III.
log w_m	9.55225	9.96317	0.37409	0.78501	1.19593		
log π	0.49715	0.49715	0.49715	0.49715	0.49715		
log V	3.39230	3.39230	3.39230	3.39230	3.39230		
Volume	2467.7	2467.7	2467.7	2467.7	2467.7		
$z=1100$	(1)	(2)	(3)	(4)	(5)	(6)	
log ϖ	1.25746	1.05200	0.84654	0.64108	0.43562	0.23016	
$\varpi_n^2-\varpi_{n+1}^2$	200.24	77.734	30.176	11.7157	4.5482		
log	2.30155	1.89061	1.47966	1.06877	0.65784		
log w_m	0.59364	1.00456	1.41548	1.82640	2.23732		
Volume	2468.0	2467.8	2467.6	2467.8	2467.8		

Since log w plots on a straight line, the mean vertical velocity for a given area between ϖ_n and ϖ_{n+1} is found by taking log $w_m=\frac{1}{2}$ (log w_n+ log w_{n+1}). It is seen that the air is ascending in the vortex at a rate of 2467.7 cubic meters per second thru each of the vortex tube rings.

THE PRESSURES IN THE VORTEX.

It is inferred, by comparing the general equations of motion, 6 and 8, that the pressure changes can be determined as follows: On any given section at the level z the third equation need not be considered, because there is no integration in a vertical direction, dz, and the second equation can be omitted, since $-\frac{1}{\rho}\frac{\partial P}{\varpi\partial\varphi}=0$, so that there remains only the first equation. The partial differentials $u\frac{\partial u}{\partial\varpi}+w\frac{\partial u}{\partial z}$ can be neglected in this vortex where the radial velocity changes slowly, except very near the cloud base; $\frac{\partial u}{\partial t}=0$ in steady motion, and $ku=0$, practically, so that there remains for computation only,

$$-\frac{1}{\rho}\frac{\partial P}{\partial\varpi}=-\frac{v^2}{\varpi}-2n\cos\theta\,.\,v,$$

that is, the centrifugal and the deflecting force.

This computation is summarized in Table 11, Section I, which contains the log $\frac{v^2}{\varpi}$, and $\frac{v^2}{\varpi}$, derived from Table 7, Section II, and Table 6, Section II, the centrifugal force being exprest in mechanical units. Since the largest value of $2n\cos\theta\,.\,v=0.0227$, this term can be neglected.

In integrating for the pressure, we have,

$$P_n-P_{n+1}=\rho_m\left(\frac{v^2}{\varpi}\right)_m(\varpi_n-\varpi_{n+1}).$$

The difference of pressure between successive rings ϖ_n, ϖ_{n+1} is equal to the mean density of the air at the elevation of the horizontal section z, multiplied by the mean centrifugal force from one ring to the other, multiplied by the distance from one ring to the other. Since the air density is not really known across the section, I can only take the mean density at the elevation z, tho it is not entirely correct and evidently too large. The mean centrifugal force $\left(\frac{v^2}{\varpi}\right)_m$ is easily found. Table 11, Section I, shows that on the same level the difference of the logs of the $\frac{v^2}{\varpi}$ is $+3\log\rho=3\times 0.20546=0.61638$. The successive values of these logs plot on a straight line so that the mean $\left(\frac{v^2}{\varpi}\right)_m$ between the rings ϖ_n and ϖ_{n+1} equals the mean of the logarithms. These values are given in Section II, together with the log ρ, which has been taken as log ρ_m.

TABLE 11.—*Computation of the pressure* B_n-B_{n+1} *thru equation C. R. 165$_1$, or S_1.*

I.—CENTRIFUGAL FORCE $\frac{v^2}{\varpi}$.

z	(1)	(2)	(3)	(4)	(5)	(6)
0	....	....	....	....	....	....
10	8.36967	8.98605	9.60243	0.21881	0.83519	1.45157
50	9.41811	0.03449	9.65087	1.26725	1.88363	2.50001
100	9.86967	0.48605	1.10243	1.71881	2.33519	2.95157
300	0.58535	1.20173	1.81811	2.43449	3.05087	3.66725
500	0.91811	1.53449	2.15087	2.76725	3.38363	4.00001
700	1.13732	1.75370	2.37008	2.98646	3.60284	4.21922
900	1.30103	1.91741	2.53379	3.15017	3.76655	4.38293
1100	1.43174	2.04812	2.66450	3.28088	3.89726	4.51364
0	....	....	....	....	....	....
10	0.023	0.097	0.400	1.66	6.84	28.3
50	0.262	1.083	4.476	18.50	76.49	316.2
100	0.741	3.062	12.660	52.34	216.37	894.5
300	3.849	15.918	65.783	271.95	1124.3	4647.8
500	8.181	34.237	141.54	585.13	2419.0	10000.
700	13.719	56.715	234.47	969.30	4007.2	16566.
900	20.000	82.682	341.82	1413.10	5841.9	24151.
1100	27.023	111.720	461.85	1909.30	7893.3	32632.

$2n\cos\theta\,.\,v$ can be neglected. For [1100, (6)] $2n\cos\theta\, v=0.0227$.

TABLE 11.—Continued.

II.—LOG OF THE MEAN $\left(\frac{v^2}{\varpi}\right)_m$.

z	(1)-(2)	(2)-(3)	(3)-(4)	(4)-(5)	(5)-(6)	log ρ_m
0	....	....	....	....	....	0.04827
10	8.67786	9.29424	9.91062	0.52700	1.14338	0.04877
50	9.72630	0.34268	0.95906	1.57544	2.19182	0.05077
100	0.17786	0.79424	1.41062	2.02700	2.64338	0.05326
300	0.89354	1.50992	2.12630	2.74268	3.35906	0.06324
500	1.22630	1.84268	2.45906	3.07544	3.69182	0.07322
700	1.44551	2.06189	2.67827	3.29465	3.91103	0.08320
900	1.60922	2.22560	2.84198	3.45836	4.07474	0.09319
1100	1.73993	2.35631	2.97269	3.58907	4.20545	0.10318

III.—PRESSURE $P_n - P_{n+1} = \rho_m \left(\frac{v^2}{\varpi}\right)_m (\varpi_n - \varpi_{n+1})$.

z	(1)-(2)	(2)-(3)	(3)-(4)	(4)-(5)	(5)-(6)
0	....	....	....	....	....
10	3.81	9.8	25.3	65.1	167.8
50	19.14	49.3	127.0	327.1	842.7
100	38.51	99.2	255.5	658.1	1695.2
300	118.2	304.5	784.2	2020.1	5203.6
500	201.6	519.2	1337.4	3445.1	8874.0
700	288.8	743.9	1916.1	4935.4	12713.0
900	379.9	978.6	2520.7	6492.7	16724.0
1100	475.1	1233.9	8152.6	8102.6	20918.0

IV.—PRESSURE $B_n - B_{n+1} = (P_n - P_{n+1}) \times 0.0075$ (in mm).

z	(1)-(2)	(2)-(3)	(3)-(4)	(4)-(5)	(5)-(6)
0	....	....	....	....	....
10	0.029	0.07	0.2	0.5	1.3
50	0.144	0.37	1.0	2.5	6.3
100	0.289	0.74	1.9	4.9	12.7
300	0.887	2.28	5.9	15.2	39.0
500	1.512	3.89	10.1	25.8	66.6
700	2.166	5.58	14.4	37.0	95.4
900	2.849	7.34	18.9	48.7	125.4
1100	3.564	9.18	23.7	60.9	156.9

The resulting difference of pressure between successive rings, at each successive elevation, is given in Section III, and the corresponding pressure differences in millimeters of mercury in Section IV. Hence, on the level $z = 1100$ meters, near the surface of the water, we have,

ϖ_6	1.7	ϖ_5	**2.7**	ϖ_4	4.4	ϖ_3	7.0	ϖ_2	11.3	ϖ_1	**18.1**
B_6	509 0	B_5	**665.9**	B_4	726.8	B_3	750 5	B_2	759 7	B_1	**763.3**

so that the difference of pressure between ring ϖ_5 and ring ϖ_1 is equal to *97.4* mm. = *3.835* inches of mercury. By the thermodynamic computations on the waterspout summarized in Table 51, MONTHLY WEATHER REVIEW, August, 1906, it was found that the difference of pressure between the cloud base and the sea level is *91.3* mm. = *3.595* inches of mercury. It is not too much to suppose that this difference, $97.4 - 91.3 = 6.1$ mm., is due to two causes, (1) an imperfection in the value of the density $\log \rho_m = 0.10318$, which should probably be taken less in the interior of the vortex than on the outside; and (2) the fact that the inner ring of the vortex which actually develops in nature may not exactly coincide with $\varpi_5 = 2.7$. That is, the central calm may not be exactly 5.4 meters in diameter. Indeed, the solution of the equations for Bessel's functions,

$$\frac{\partial^2 \varphi}{\partial \varpi^2} - \frac{1}{\varpi}\frac{\partial \varphi}{\partial \varpi} + a^2 \varphi = 0\,,$$

which can be derived from the vortex equation,

$$\frac{\partial^2 \psi}{\partial \varpi^2} - \frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi} + \frac{\partial^2 \psi}{\partial z^2} = 2\,\varpi\,\omega_2\,,$$

results in a root, $a\,\varpi_0 = 3.832$. It is probable that $a = 1$, and it has been taken as unity in the formulas for this waterspout, so that $\varpi_0 = 3.832$, which is the radius of the closest vortex tube to the axis. My computation carried the development to $\varpi_6 = 1.70$ meters, but it should probably stop short of $\varpi_5 = 2.7$, tho at what point it is not possible to decide. *We may conclude that the innermost pressure of the vortex at the sea level is about equal to that at the cloud level from whence the vortex was projected.* This view can be strengthened by the following consideration. In a pure vortex of this type the rotating velocity next to the calm core at any level is apparently equal to that of a body falling freely from the plane of reference thru the distance z, so that $v^2 = 2\,gz$.

TABLE 12.—*Comparison of v_5 with v and q.*

Comparison of $v = \sqrt{2\,gz}$ with v_5 in Table 7, Section II.					Comparison of v_5 with $q = 23.06 \sqrt{T.\Delta B/B}$.
z	log $2g\,z$	log v	v	v_5	From formula 13, page 470, Monthly Weather Review, October, 1906:
0			0.0	0.0	$q = 23.06 \sqrt{\frac{T}{B}\,\Delta\,B}.$
10	2.29252	1.14626	14.0	14.0	
50	2.99149	1.49575	31.3	31.3	For $\begin{cases} T = 292.7°, \text{Table 51.} \\ \Delta\,B = 97.4^{mm}, 763.3 - 665.9 = B_1 - B_5. \\ B = 763\ 3^{mm}, \text{Table 51.} \end{cases}$
100	3.29252	1.64626	44.3	44.2	
300	3.76964	1.88482	76.7	76.6	
500	3.99149	1.99575	99.0	98.9	$v_5 = q = 141.0$ meters per second.
700	4.13762	2.06881	117.2	117.0	
900	4.24676	2.12338	132.9	132.7	
1100	4.33391	2.16696	146.9	146.7	

The close agreement between the value of the falling velocity, $v = \sqrt{2gz}$, and v_5, the velocity on the edge of the core, as given in Table 7, Section II, seems to indicate that this is a possible way in which to begin the discussion of such vortices in the atmosphere, or at least to check the results, as in this instance.

By plotting the points in a curve indicated by the coordinates (ϖ, $B_n - B_{n+1}$), as given in Table 6, II, for the radial distance ϖ, and in Table 11, IV, for the differences of the pressure between successive rings, it is found that they form a logarithmic curve, and consequently the logarithms of these coordinates plot on a straight line. The computation shows that

$$\log \frac{(B_n - B_{n+1})}{(B_{n-1} - B_n)} = 2 \log \rho \qquad = 2 \times 0.20546 = 0.41092,$$

$$\log \frac{\varpi_n}{\varpi_{n+1}} = \log \rho \qquad = \qquad = 0.20546,$$

Hence, $$\log \frac{(B_n - B_{n+1})}{(B_{n-1} - B_n)} - \log \frac{\varpi_n}{\varpi_{n+1}} = \log \rho \qquad = 0.20546,$$

so that the logarithmic relation between the spaces within the successive vortex tubes and the corresponding pressures is thus determined. This value of $\log \rho = \log \frac{\varpi_n}{\varpi_{n+1}} = 0.20546$ is fundamental to the structure of a vortex, and it seems to be an atmospheric constant which should be carefully determined.

RELATION OF THE TEMPERATURE TO THE VORTEX MOTION.

The thermodynamic energy which generated this waterspout may be attributed to two principal sources. The first is the vertical rise of the lower strata induced by the general cloud motion and due to the overflowing cold stratification. The cloud generally rises in the central portions and falls on the edges, and this upward buoyancy is converted from a broad surface at the cloud base into a narrow vortex tube, *wherein the cloud surface descends in a small area to the sea level.* The second source of energy is the horizontal pressure flow of two strata of different temperatures, so that the pressure shall remain the same on each side of the surface of discontinuity. This subject will be taken up at length in the later papers of the series, but it may be noted here that the following relation holds:

First stratum: $$-\int \frac{dP_1}{\rho_1} = \tfrac{1}{2}\left(v_1^2 - v_0^2\right) + g\left(z_1 - z_0\right).$$

Second stratum: $$-\int \frac{dP_2}{\rho_2} = \tfrac{1}{2}\left(v_2^2 - v_0^2\right) + g\left(z_2 - z_0\right).$$

In order that on the same boundary, where $z_1 = z_2$, the pressure shall be the same, $P_1 = P_2$, after subtracting there will remain,

$$\rho_1 \tfrac{1}{2} (v_1^2 - v_0^2) = \rho_2 \tfrac{1}{2} (v_2^2 - v_0^2).$$

From the general law,

$$P_1 = \rho_1 RT_1.$$
$$P_2 = \rho_2 RT_2.$$

For $P_1 = P_2$, $\rho_1 = \frac{\rho_2 T_2}{T_1}$. Substituting,

$$\rho_2 \frac{T_2}{T_1}(v_1^2 - v_0^2) = \rho_2 (v_2^2 - v_0^2). \quad \text{Hence,}$$

$$T_2 (v_1^2 - v_0^2) = T_1 (v_2^2 - v_0^2).$$

The relative velocity of one stratum, $(v_1^2 - v_0^2)$, multiplied by the temperature of the second, T_2, equals the relative velocity of the second stratum, $(v_2^2 - v_0^2)$, multiplied by the temperature of the first, T_1; and this maintains the pressure as if the air had no motion, and the temperature gradients remained normal. The first type of vortex with the funnel-shaped tube depends upon the first principle more than upon the second, while the second type of vortex with the dumb-bell tube depends upon the second rather than upon the first. This will be illustrated by the Chamberlain 2d A, the St. Louis tornado, and the De Witte hurricane. The ocean cyclone has in addition to these two sources of motion a third, similar to the last, but modified by the fact that the boundary of the stratification between the cold and warm masses instead of being horizontal is vertical in part, as shown by the temperature distributions in cyclones and anticyclones up to 10,000 meters. The land cyclones depend more decidedly upon the third source of motion than does the ocean cyclone.

II.—THE THEORY OF VORTEX MOTION APPLICABLE TO THE DUMB-BELL-SHAPED TUBE IN THE COTTAGE CITY WATER-SPOUT.

THE DUMB-BELL-SHAPED TYPE, COTTAGE CITY WATERSPOUT, CHAMBERLAIN 2D A.

An examination of the photographs of the Cottage City waterspout given in the MONTHLY WEATHER REVIEW for July, 1906, pp. 307–315 and Plates I–X, shows that two distinct forms of the tube or types of the vortex were developed at different times from the same cloud. At the second appearance, 1:02 p. m. to 1:17 p. m. (Plates I–VII), the dumb-bell-shaped type prevailed (see Chamberlain's photograph 2d A); and at the third appearance, 1:20 p. m. to 1:27 p. m. (Plates VIII–X.), the funnel-shaped type was exhibited. In all accessible photographs of tornadoes these two types occur quite indifferently in numbers, apparently developed by subtle differences in the physical conditions of the cloud at the several occasions of their formation. While both types are of theoretical interest, it is much more important for the meteorologist to understand the dumb-bell type, because the large tornadoes, the hurricanes, and the cyclones in part, are constructed upon the same principles, differing from one another only in their dimensions and proportions. Since the ultimate explanation of the motions of the atmosphere in cyclones and anticyclones seems to be very closely associated with the theory of dumb-bell vortices, it will be proper to keep in mind the goal toward which this present exposition tends.

It can easily be seen in the photographs above referred to, 2d A to 2d G, inclusive (Plates I to VII), that the tube, instead of continuing to taper from the cloud to the sea level, reaches a minimum diameter more than halfway down from the cloud to the sea and then begins to expand. The lower portion is not entirely visible, on account of the enveloping cascade of spray, and it will be shown in these papers that, in fact, the lowest section is not fully developed, and that the vortex tube is amputated or truncated by the sea-level surface at from one-twentieth to one-third of its theoretical length, according to circumstances. The corresponding upper section is fully developed at the cloud, tho the tube and the cloud merge into one another before the asymptotic extension of the vortex is reached. When the tube begins to break up, and the gyratory velocity diminishes, the dumb-bell form appears more clearly, as on 2d F, and it is very distinct on 2d G. In the earlier numbers of the series, 2d A to 2d E, the inner tubes of the complete vortex, which have very great velocities, are formed, but the outer tubes appear as the rotation velocity falls in amount.

According to the formulas of the first paper of this series (compare Table 3 and Cloud Report, 1898, page 513), we begin with the vortex system exprest as follows:

1. Current function. $\psi = \frac{v\varpi}{a} = A\varpi^2 \sin az.$
2. Radial velocity. $u = -\frac{1}{\varpi}\frac{\partial \psi}{\partial z} = -Aa\varpi \cos az.$
3. Tangential velocity. $v = \frac{a\psi}{\varpi} = Aa\varpi \sin az.$
4. Vertical velocity. $w = \frac{1}{\varpi}\frac{\partial \psi}{\partial \varpi} = 2A \sin az.$

APPLICATION OF THE FORMULAS TO THE COTTAGE CITY WATERSPOUT, CHAMBERLAIN 2D A.

The primary difference between the funnel-shaped and the dumb-bell-shaped vortex tube is that the former extends from its asymptotic relation at one plane of reference, in the base of the cloud, perpendicularly to a great distance from it, tapering continuously to a tube of very small dimensions, while the latter becomes asymptotic to two planes of reference, one in the cloud base and the other at or below the surface of the sea. Not only is the distance between the two reference planes to be measured in meters, but the axis or connecting line is also to be divided into 180 parts or degrees. Thus, in Fig. 3, assume that the upper line is 1200 meters from the lower line, that the axis is of the same length, and that this represents the entire vortex. If this length is taken as 180° or parts then the a appearing in the formulas is

$$a = \frac{180}{1200} = 0.150 \ [9.17609],$$

which gives the angular change per meter. Since the symmetry of the formulas, as controlled by the sine and cosine terms, shows that the variations lie between $+1$ and -1, it follows that $\sin az$ and $\cos az$ will carry the function thru all the intermediate values. Fig. 3 is constructed by plotting the lines determined by the coordinates of Table 17, which gives the radii ϖ of the several tubes at different heights z.

Since there is no way to determine the value of the tangential velocity at any given point, it is necessary to assume a value for v at a point (ϖ, z). The correctness of the one adopted can be checked by constructing the vortex from these data, and comparing it with the shape as shown on the photograph. The height z was determined as about 1200 meters by the measurements, and after several trials I have taken

$$az = 170° \text{ or } 10°,$$
$$\varpi = 200 \text{ meters},$$
$$v = 2 \text{ meters per second}.$$

The value $az = 10°$ is for a point near the sea level, and the value $az = 170°$ is for a point just below the cloud base. Hence we have the current function,

$$a\psi = v\varpi = 400 \ [2.60206].$$

For the value of the ratio of the successive radii, at the points separated by 10-millimeter intervals of pressure, as 760, 750 ...... 690, we shall assume the same value as that given on page 469, whose logarithm is,

$$\log \rho = 0.20546.$$

These data enable us to proceed with the computations in the regular order, and to develop the entire structure of this